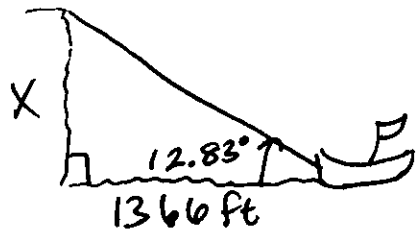
3.7 Solving Problems with Trig Key 75 pts.

Name_____ Date _____ Period _____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

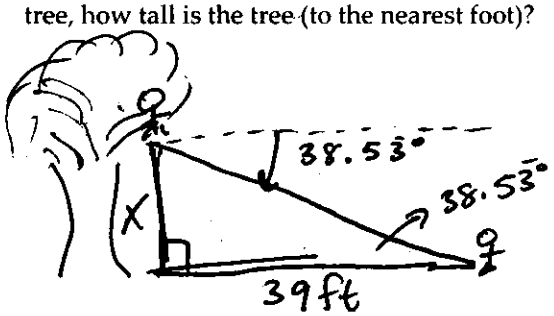
2 1) From a boat on the lake, the angle of elevation to the top of a cliff is 12°50'. If the base of the cliff is 1366 feet from the boat, how high is the cliff (to the nearest foot)? 1) _____



$\tan 12.83° = \frac{x}{1366}$

$x = 1366 \cdot \tan(12.83°) \approx$ 311 ft

2 2) When sitting atop a tree and looking down at his pal Joey, the angle of depression of Mack's line of sight is 38°32'. If Joey is known to be standing 39 feet from the base of the tree, how tall is the tree (to the nearest foot)? 2) _____

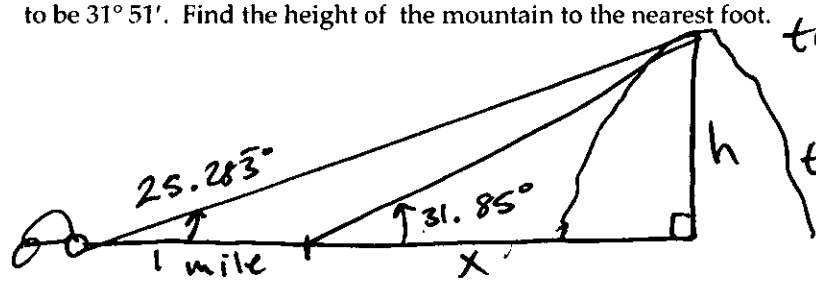


$\tan 38.5\bar{3}° = \frac{x}{39}$

$x = 39 \cdot \tan(38.5\bar{3}°)$

$x \approx$ 31 ft

2 3) Bob is driving along a straight and level road straight toward a mountain. At some point on his trip he measures the angle of elevation to the top of the mountain and finds it to be 25° 17'. He then drives 1 mile (1 mile = 5280 ft) more and measures the angle of elevation to be 31° 51'. Find the height of the mountain to the nearest foot. 3) $h \approx 10,406$ ft.



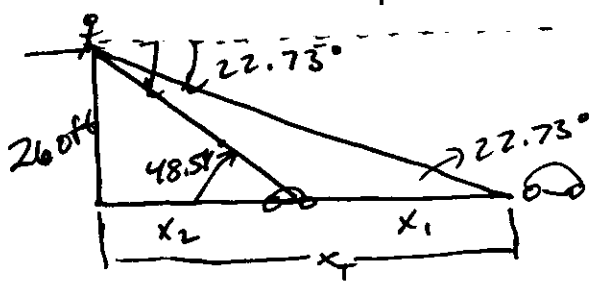
$\tan 25.28\bar{3}° = \frac{h}{x+5280} \rightarrow h = (x+5280)\cdot \tan(25.28\bar{3})$

$\tan 31.85° = \frac{h}{x} \Rightarrow h = x\tan(31.85°)$

$(x+5280)(\tan 25.28\bar{3}) = x\tan(31.85)$

solve for x. $x \approx 16738$, then h.

2 4) A person is watching a car from the top of a building. The car is traveling on a straight road directly toward the building. When first noticed the angle of depression to the car is 22° 44' . When the car stops, the angle of depression is 48° 36' . The building is 260 feet tall. How far did the car travel from when it was first noticed until it stopped? Round your answer to the hundredths place. 4) _____



$\tan 22.73° = \frac{260}{x_T}$, $x_T = \frac{260}{\tan 22.73°} = 620.84$

$\tan 48.6° = \frac{260}{x_2}$, $x_2 = \frac{260}{\tan 48.6°}$

$x_2 = 229.22$

$x_T = x_1 + x_2$

$x_1 = x_T - x_2$

1 $x_1 = 620.84 - 229.22$

$= 391.62$ ft (rounding?)

391.41 ft

5) The number of hours of darkness in a coastal town can be modeled by $f(x) = 6.1 \cos\left[\frac{\pi}{6}(x - 2)\right] + 12.1$, where x is the month and x = 1 corresponds to January. 5) _____

2 Approximate the number of hours of darkness in April, to the nearest tenth of an hour.

$f(4) = 6.1 \cos\left(\frac{\pi}{6}(4-2)\right) + 12.1$

$= 15.2$ hrs

6) The position of a weight attached to a spring is $s(t) = -4 \cos 5t$. What are the frequency and period of the system? 6) _____

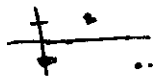
2 $b = 5$ period: $\frac{2\pi}{5}$

frequency: $\frac{5}{2\pi}$

7) The position of a weight attached to a spring is $s(t) = -6 \cos 16\pi t$ inches after t seconds. What is the maximum height that the weight reaches above the equilibrium position and when does it first reach the maximum height? 7) _____

2 max height 6 inches.

at $t = \frac{1}{16} = .0625$



period $\frac{2\pi}{16\pi} = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$

8) A Ferris wheel 50 feet in diameter makes one revolution every 40 seconds. The center of the wheel is 30 feet above the ground. Write a cosine function to model the height of a car on the Ferris wheel at any time t. 8) _____

2 $h(t) = -25 \cos\left(\frac{\pi}{20}t\right) + 30$

$r = 25$

$\frac{1 \text{ rev}}{40 \text{ sec}} = \text{frequency}$

$\text{period} = \frac{40 \text{ sec.}}{1}$

$\frac{2\pi}{b} = 40$ $\quad b = \frac{\pi}{20}$

9) A buoy oscillates up and down as waves go past. The buoy moves a total of 4.2 feet from its low point to its high point, and then returns to its high point every 10 seconds. Write a cosine function modeling the bouy's vertical position at time t. 9) _____

2 $a = 2.1$ ft

$Y = -2.1 \cos\left(\frac{\pi}{10}t\right)$

$p = 20$

$20 = \frac{2\pi}{b}$ $\quad b = \frac{2\pi}{20} = \frac{\pi}{10}$

10) Low tide is at 11:15 am and high tide is at 5:15 pm. The water level varies 68 inches between low and high tide. Write a cosine function to represent the change in water level. 10) _____

2 $f(t) = -34 \cos\left(\frac{\pi}{6}t\right)$

1 period = 12 hrs.

$12 = \frac{2\pi}{b}$

$b = \frac{2\pi}{12} = \frac{\pi}{6}$

Find all solutions on the interval $[0, 2\pi)$.

2 11) $\sin x = -\frac{\sqrt{2}}{2}$ $x = 5\pi/4, 7\pi/4$ 11) _____

2 12) $\cos x = 1/2$ $x = \pi/3, 5\pi/3$ 12) _____

2 13) $\tan x = -1$ $x = 3\pi/4, 7\pi/4$ 13) _____

2 14) $\sec x = -2/\sqrt{3}$ $x = 5\pi/6, 7\pi/6$ 14) _____

2 15) $\csc x = 2$ $x = \pi/6, 5\pi/6$ 15) _____

2 16) $\cot x = -1$ $x = 3\pi/4, 7\pi/4$ 16) _____

Use your calculator to find all solutions in the interval $[0, 2\pi)$.

2 17) $\sin x = 0.33$ $x = 0.336$ & $x = 2.806$ 17) _____

$\sin^{-1}(0.33) =$

2 18) $\cos x = 0.59$ $x = 0.93$ & $x = 5.35$ 18) _____

2 19) $\tan x = 1.615$ $x = 1.016$ & $x = 4.158$ 19) _____

2 20) $\cos x = -0.36$ $x = 1.939$ & $x =$ ~~5.340~~ 4.344 20) _____

Find all solutions in the interval $[0, 2\pi)$ for each trigonometric equation.

2 21) $2\sin x + \sqrt{3} = 0$ $\frac{2\sin x}{2} = \frac{-\sqrt{3}}{2}$ $\sin x = -\frac{\sqrt{3}}{2}$ 21) $x = 4\pi/3, 5\pi/3$

2 22) $3\tan x - \sqrt{3} = 0$ $3\tan x = \sqrt{3}$ 22) $x = \pi/6, 7\pi/6$

$\tan x = \frac{\sqrt{3}}{3}$

3 23) $\cos x + \sqrt{2} = -\cos x$

$2\cos x = -\sqrt{2}$

$\cos x = \frac{-\sqrt{2}}{2}$

23) $x = 3\pi/4, 5\pi/4$

3 24) $4\tan x = 3 + \tan x$

$-\tan x$

$3\tan x = 3 \quad \tan x = 1$

24) $x = \pi/4, 5\pi/4$

5 25) $2\cos x \sin x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \rightarrow x = \pi/2, 3\pi/2$

$2\sin x - 1 = 0$

$\sin x = 1/2 \rightarrow x = \pi/6, 5\pi/6$

25) _____

5 26) $4\cos^2 x - 1 = 0$

$(2\cos x + 1)(2\cos x - 1) = 0$

$\cos x = -1/2 \quad \cos x = 1/2$

$x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

26) _____

4 27) $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = -1/2 \quad \sin x = -1$

$x = 7\pi/6, 11\pi/6, 3\pi/2$

27) _____

3 28) $4\cos^2 x - 4\cos x + 1 = 0$

$(2\cos x - 1)(2\cos x - 1) = 0$

$\cos x = 1/2$

$x = \pi/3, 5\pi/3$

28) _____

3 29) $2\sin^2 x - 3\sin x = 2$

$2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$\sin x = -1/2 \quad \sin x = 2$

$x = 7\pi/6, 11\pi/6$

29) _____

5 30) $\tan x \sin^2 x = \tan x$

$\tan x \sin^2 x - \tan x = 0$

$\tan x(\sin^2 x - 1) = 0$

$\rightarrow (\sin x + 1)(\sin x - 1)$

$\tan x = 0 \quad \sin x = -1 \quad \sin x = 1$

$x = 0, \pi, 3\pi/2, \pi/2$

30) _____